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ON THE PAPER BY KIRICHENKO V.F., KRYS'KO V.A. AND SUROVA N.S.: THE BUBNOV-GALERKIN METHOD IN THE NON-LINEAR THEORY OF HOLLOW FLEXIBLE, ORTHOTROPIC MULTILAYERED SHELLS*

D.G. PERADZE

There are a number of errors in the paper by Kirichenko et al when proving the existence of a solution and convergence of the approximate method for the system of differential Timoshenko equations for a shell.

The authors studied the problem of the existence, in the region $\Omega \subset E_2$, of a solution and of the convergence of the Bubnov-Galerkin method for a system of non-linear differential equations describing the behaviour of a shell in the Timoshenko model, with Dirichlet-type boundary conditions. The system of equations with unknown functions $u, v, w, \gamma_x, \gamma_y$ is supplemented by a term containing the coefficient ε biharmonic in w. This produces an auxiliary problem which is then used, for any $\varepsilon > 0$, to show that the auxiliary problem has a solution and that its approximate solution can be obtained using the Bubnov-Galerkin method. A passage to the limit as $\varepsilon \to 0$ is then carried out.

Before all that the a priori estimates for the ε-solution are derived

$$|u_\varepsilon|_{01} \leqslant c, \ |v_\varepsilon|_{01} \leqslant c, \ |\sqrt{\varepsilon} w_\varepsilon|_2 \leqslant c, \ |\gamma_{x\varepsilon}|_{01} \leqslant c, \ |\gamma_{y\varepsilon}|_{01} \leqslant c \tag{1}$$
$$c = \text{const} > 0$$

The authors assert that, as $\varepsilon \to 0$, relations (1) imply the existence of a sequence $\{u_\varepsilon, v_\varepsilon, w_\varepsilon, \gamma_{x\varepsilon}, \gamma_{y\varepsilon}\}$ (we use the previous notation for simplicity), converging to $\omega^\circ = (u^\circ, v^\circ, w^\circ, \gamma_x^\circ, \gamma_y^\circ)$ in the sense that $u_\varepsilon \to u^\circ, v_\varepsilon \to v^\circ, \gamma_{x\varepsilon} \to \gamma_x^\circ, \gamma_{y\varepsilon} \to \gamma_y^\circ$ weakly in $W^{01}_2(\Omega)$, $w_\varepsilon \to w^\circ$ weakly in $W_2^2(\Omega) \cap W_2^{01}(\Omega)$ and $w_\varepsilon \to w^\circ$ strongly in $W_2^{01}(\Omega)$. At the same time, relations of the type

$$\iint_\Omega \left[\frac{\partial u_\varepsilon}{\partial x} - k_x w_\varepsilon + \frac{1}{2}\left(\frac{\partial w_\varepsilon}{\partial x}\right)^2\right] \frac{\partial w_\varepsilon}{\partial x} \frac{\partial \varphi}{\partial x}\, d\Omega \to$$
$$\iint_\Omega \left[\frac{\partial u^\circ}{\partial x} - k_x w^\circ + \frac{1}{2}\left(\frac{\partial w^\circ}{\partial x}\right)^2\right] \frac{\partial w^\circ}{\partial x} \frac{\partial \varphi}{\partial x}\, d\Omega,$$
$$k_x \in L_2(\Omega), \ \forall \varphi \in W_2^2(\Omega) \cap W_2^{01}(\Omega)$$

are used to deuce that ω° is a solution of the initial problem.

The conclusions would be justified if the third equation of (1) did not contain the factor $\sqrt{\varepsilon}$. The presence of this factor nullifies their validity.

Moreover, the first two eatimates in (1), as well as the third one, which is used to obtain the previous two estimates, must contain the factor $\sqrt{\varepsilon}$, i.e. it must have the form

$$|\sqrt{\varepsilon} u_\varepsilon|_{01} \leqslant c, \ |\sqrt{\varepsilon} v_\varepsilon|_{01} \leqslant c$$

which causes additional difficulties.

The authors are incorrect in their arguments and hence the proof of the fundamental result of the paper (Theorem 2) is false.

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